Report on the SEAMS School "Module Theory and Its Applications"

in Bandung, Indonesia, November 10th – November 18th, 2014.

Subject

The subject was chosen to introduce many aspects on Module Theory: modules as generalization of vector spaces and properties that still hold or need to be modified, the notion of prime submodules as generalization of prime ideals, representation of modules given by quiver representations and the application of modules over finite rings in Coding Theory. The presence of a large number of mathematicians working in Module Theory is going to strengthen the impact of this research discipline in Indonesia. The students come from many parts of the greater region and we intend to use the contacts to build on further networks centering on Module Theory in South-East Asia. Further we use the leading role the mathematical faculty of ITB to advertise the subject in Indonesia more than it is already.

The participants

Beside Indonesian participants, 11 participants from Pakistan, Cambodia, Vietnam, Thailand, Myanmar, Nepal, Philippines and Mexico were chosen. The students who are eager to participate and having knowledge in the subject, and therefore having a real chance to follow the lessons, were accepted. All participants, with some very few exceptions have followed the entire lessons.

Two participants from Myanmar cancelled their participations just two days before the school started because as government official, they did not get permission from their government to leave the country. One participant from Cambodia cancelled his participation because of health problem.

Finally there were 8 participants from abroad: 1 from Pakistan, 2 from Vietnam, 2 from Thailand, 1 from Nepal, 1 from Philippines and 1 from Mexico.

The Indonesian participants also came from various regions: 2 from Jakarta, 1 from Lampung, 6 from Yogyakarta, 3 from Lombok, 1 from Bali, 1 from Kalimantan and 1 from Makassar, 6 just graduated from ITB and Gadjah Mada University.

Contents of lessons by speaker

1. Irawati started with known algebraic structures such as abelian groups, rings and fields. Then she defined action from a field to an abelian group to introduce vector space and action from a ring to an abelian group to introduce module. She gave comparison between vector space and module where some vector space properties doesn't hold for module. Then she defined substructures and mappings to compare two systems.

   At her second session, she gave some overview on Research in Module Theory. Started with property of principal ideal domain where every ideal is invertible, she defined a Dedekind domain as a domain where every ideal is invertible. Next, she generalized it for noncommutative

ring and defined Dedekind prime ring, then explained its relation with Hereditary Noetherian Prime Ring (HNP Ring). Then she continued with generalization of prime ring into M-prime module, Noetherian ring into M-Noetherian module, and hereditary ring into M-hereditary module. She also explained about the decomposition property of finitely generated module over PID that is also satisfied for finitely generated module over HNP ring. Finally she defined Dedekind module, Bezout module, p-Bezout module and n-Bezout module as generalization of Dedekind prime ring, Bezout ring, p-Bezout ring and n-Bezout ring, respectively.

2. Sri Wahyuni started with more detailed explanation on how the idea of module comes up as generalization of vector space, why we differ between left and right modules, how to construct a module from an abelian group. Then she defined quotient module using the idea that quotient group can be constructed over a normal subgroup and quotient ring over an ideal. Next she explained how to construct a submodule from a given subset of a module, she gave the idea of generators, and ended the first session by the idea of free module as module does not always have basis.

   For the second part Sri Wahyuni explained about module homomorphism and Fundamental Theorem of Module Homomorphism and its applications. Then she defined the set of all homomorphisms between two modules, and that set has group structure with addition. In particular the set of all endomorphisms has ring structure. Next she explained that the set Hom(M,M') has a module structure over End(M). She ended the session by recalling free module and its properties, module as quotient of free module, and the relation between split exact sequence and free module to give motivation for defining projective module.

3. Hanni Garminia continued Sri Wahyuni's presentation by characterizing rings where every projective module is free. She continued with modules over PID and gave more detailed results on Dedekind modules and HNP modules. She explained one result on Module Theory research: about quotient module of a Dedekind module. Finally she ended the session by giving open problems in Module Theory research: first how to define Dedekind module over non commutative rings and second how to define Dedekind comodules.

4. Nguyen Van Sanh started with the notion of prime submodule and described all its properties as generalization of prime ideal. Then he defined the notion of strongly prime submodule as generalization of the notion of prime ideal over a commutative ring. Next he defined prime module and semiprime module, then gave a relation between prime ring and prime module: endomorphism ring of prime module is a prime ring and a self-generator module which endomorphism ring is a prime ring is a prime module. He also gave similar property for semiprime module but with condition that the module should be quasi-projective finitely generated and self-generator.

   For the second session Nguyen Van Sanh explained a result concerning prime submodules: generalization of Kaplansky-Cohen Theorem on modules. The theorem said that in order to check whether every ideal in a commutative ring is finitely generated (principal), it is enough to

check prime ideals. For module case he explained the following result: Let M a quasi-projective finitely generated R-module which is a self-generator. If M is Noetherian as End(M)-module and also as module-R, and if every maximal submodule of M is M-cyclic, then every submodule of M is M-cyclic.

As the final session Nguyen Van Sanh explained about Nil radical and Levitzki radical of modules and their properties, as generalization of Nil radical and Levitzki radical of rings. He also gave some properties concerning the relation between P(M) (the intersection of all prime submodules of M), L-rad(M) (Levitzi radical of M), Nil(M) and Rad(M).

5. Indah Wijayanti started by defining coalgebra over a commutative ring, cocommutative coalgebra and coalgebra morphism. Then she defined comodule over coalgebra, the category of right comodules, subcomodule and subcoalgebra. Next she explained how to view comodule over coalgebra C as module over C*, the dual of C. Related to Nguyen Van Sanh's topic, given a coalgebra C, Indah Wijayanti defined prime right C-comodule as prime left C*-module. And then she gave characterizations of prime comodule and relation between prime ring and prime comodule. She also defined coprime C-comodule as a coprime C*-module.

   For the second session Indah Wijayanti started by recalling definition of prime ring, generator, cogenerator, and prime module and its characterizations. Then she gave properties of prime module related to prime ring, M-cogenerated module and projective module. Then she defined coprime module and gave its characterizations, its relation with prime ring and some examples. Next she defined endo-prime module as a module M which is a prime End(M)-module. She also gave properties of endo-prime module related to prime ring. And finally she defined endo-coprime module and its properties.

6. Intan Muchtadi started by defining algebra over field K as a ring that has structure of K-vector space which compatible with the ring structure. Then she defined subalgebra, ideal of an algebra and K-algebra homomorphism. Next she defined Jacobson radical of a K-algebra and gave a method to find the radical. She continued by defining module over algebra, submodule, finitely generated module, module homomorphism, module isomorphism, idempotent, and complete set of primitive orthogonal idempotents.

   At the second session, she defined basic algebra, gave its properties and the Morita equivalence between an algebra and its basic algebra. Then she defined quiver, path, cycle, loop, acyclic quiver, and the path algebra as algebra which basis consists of paths. Next she gave properties of path algebra: associativity, identity element, a necessary and sufficient condition to become a finite dimensional algebra, and when it will be connected. Then she defined arrow ideal and explained that the arrow ideal is the radical of the path algebra if the quiver is acyclic.

   For the last session, she defined admissible ideal, bound quiver algebra and gave some examples. Then she defined relation, and quiver bound by relation; and she gave properties of bound quiver algebra: connectedness, finite dimensional, its radical and explained why it is a basic algebra. Next she explained how to construct a quiver from a basic connected finite

dimensional algebra, and how the correspondence between finite connected quivers and basic connected finite dimensional algebras.

7. Dellavitha Nasution started by defining representation of quiver, morphism between quiver representations and the category of quiver representation, and she gave some examples. Then she explained the equivalence of category between the category of module over a bound quiver algebra KQ/I and the category of representation of Q. Therefore this gives a way to get a visualization of module.

   At the second session she explained the quiver representation of simple module over a bound quiver algebra and how to identify semisimple module from the quiver representation. Next she gave quiver representation of radical, socle and top of a module. Finally she gave quiver representation of projective and injective module.

8. Aleams Barra started with elementary coding theory: the coding terminology associated to error-correcting codes, how to detect and correct errors. Then he gave real application of coding theory for example for ISBN, mariner mission and compact disc.

   For the second session Aleams Barra explained about Algebraic Coding Theory, how we need good alphabet with a rich structure to write a message, so we use finite field; and encoding becomes a linear map. He defined a linear code of a length n as a linear subspace of F^n and explained the use of generator matrix for encoding and error detection. Next he defined the Hamming distance between two vectors, the Hamming weight of a vector and the minimum distance of a code. Then he explained when two codes are equally good and how to characterized them by Macwilliams Equivalence Theorem.

   At the third session he explained two approaches to proof the MacWilliams Equivalence Theorem. The first approach is by using matrix approach, and the second one is by using character approach. The character approach is needed to get a MacWilliams Equivalence version for codes over ring.

   For the fourth session he explained about codes over finite rings, where the high development started in 1994. For a finite ring R a code over R of length n is a left R-submodule of R^n. Some properties of codes over field do not hold for codes over ring, therefore the MacWilliams Equivalence Theorem needs to be reformulated. For the proof, he explained why the character approach is needed: the rows of a generator matrix are not always linearly independent and characters are still linearly independent; then he gave the proof. He ended this session by explaining that if the MacWilliams Equivalence Theorem holds then the ring is Frobenius. Therefore Frobenius ring is the right class of ring to work with in codes over ring.

   As the last session Aleams Barra explained codes over a left R-algebra A, where R is a finite ring. A linear code of length n over A is a left submodule of A^n. Then he defined monomial transformation, weight on the alphabet A, and the left and right symmetry group of a weight,

and used Hamming weight as an example. Next he defined extension property of an alphabet A with respect to weight, and defined Frobenius bimodule. He showed that if A is a Frobenius bimodule that A has the extension property with respect to Hamming weight. He ended the session by giving a theorem by Wood, a necessary and sufficient condition for an alphabet to have extension property with respect to Hamming weight.

9. Beside the 4 to 5 hours lessons each there were 5 times group discussions each 60-90 minutes: two times for Introduction to Theory of Modules, one time for Prime Modules, one time for Module Representations, and one time for Module in Coding Theory. The participants were working enthusiastically to solve the exercises given by the speakers. After discussions in groups, one representative of each group will present the problem solutions. The schedule gave enough time for discussions between participants and between participants and the speakers. These opportunities have been used well and collaborations could be initiated.

The accommodation

The non-ITB speakers and almost all non-ITB participants were lodged in the same hotel near ITB. This gives a big opportunity for further discussions between the participants and also between the speakers and the participants.

Excursion

An excursion was organized the Saturday between the weeks of lesson. In the morning they visited a close active volcano in Bandung, and in the afternoon musicians on traditional instrument gave a concert. This was very delightful and the participants were fascinated. This made possible the gathering between the participants and the speakers except the strictly scientific frame.

Financial aspects

The research school benefited from a support on behalf of CIMPA, which covered almost 40% of the total budget, which was used for transportation and dinner cost of overseas participants, the transportation and accommodation of the overseas speaker and the airplane ticket of one Indonesian speaker. The IMU contributed the accommodations of the overseas participants. The Consortium grant of the Indonesian Algebra Society contributed the accommodation of the Indonesian participants and Indonesian speaker, the transportation of one Indonesian speaker, and local expenses. The Faculty of Mathematics and Natural Sciences ITB, some research grant and registration fee of some participants supplemented the budget.

For the one participant from Cambodia that cancelled his participation, unfortunately we already paid for his ticket and could not cancel it because there was no news from him. Finally on Wednesday November 12th, 2014 we received an email saying that he was at the hospital and could not inform us earlier. Since the students from Myanmar and Cambodia did not come, we were able to fully reimburse the airfares of the participants from Pakistan, Vietnam, Thailand, Nepal and Philippines and partially reimburse the participant from Mexico.

Source					
		EUR	EUR	Rp	**Rp**
CIMPA		4,500.00		67,500,000.00	
IMU		1,581.10		23,716,500.00	
Fac of Maths and Natural Sciences ITB (FMIPA)		1,333.33		20,000,000.00	
ITB and Asahi Research Grant		521.67		7,825,000.00	
Consortium Grant		2,893.33		43,400,000.00	
Registration from participants non-support (participants)		626.67		9,400,000.00	
Total			**11,456.10**		**171,841,500.00**
Expense					
From CIMPA					
Airplane ticket	9 Overseas Students	3,211.85		48,177,700.00	
Airport tax	8 Overseas Students	80.00		1,200,000.00	
Transportation Jakarta airport -Bandung return	1 Overseas Students	33.33		500,000.00	
Transportation Bandung airport - Hotel return	6 Overseas Students	40.00		600,000.00	
Transportation Bangkok - Bandung	1 Overseas Speaker	299.83		4,497,400.00	
Transportation Jogja -Bandung	1 Indonesian Speaker	116.67		1,750,000.00	
Visa Fee	2 Overseas Students	130.00		1,950,000.00	
10 days dinner	7 Overseas Students	233.33		3,500,000.00	
10 days accommodation	1 Overseas Speaker	300.00		4,500,000.00	
			4,445.01		**66,675,100.00**
From FMIPA					
7 days lunch	42 persons	980.00		14,700,000.00	
school dinner	35 persons	140.00		2,100,000.00	
Seminar Kit (certificates, name tag, book program, souvenirs for speakers)		213.67		3,205,000.00	
			1,333.67		**20,005,000.00**
From ITB and ASAHI Research Grant					
7 days snacks	60 persons	333.33		5,000,000.00	
Photocopy		188.33		2,825,000.00	
			521.67		**7,825,000.00**
From Consortium Grant					
Transportation Jogjakarta - Bandung return	1 Indonesian Speakers	121.91		1,828,600.00	
4 days accommodation	1 Indonesian Speaker	90.67		1,360,000.00	
2 days accommodation	1 Indonesian Speaker	45.33		680,000.00	
9 days accommodation	12 Indonesian Students	1,500.00		22,500,000.00	
7 days lunch	28 persons	666.67		10,000,000.00	
Stationery (pen, stempel, cartridge, glue, papers, banners)		102.03		1,530,500.00	
Seminar Kit (school bags)	100 persons	266.67		4,000,000.00	
School dinner	25 persons	100.00		1,500,000.00	
			2,893.27		**43,399,100.00**
From registration fee					
Operational (room maintenance, mineral water, boardmarkers)		129.70		1,945,500.00	
Excursion		78.23		1,173,500.00	
Flash disks for participants	84 persons	420.00		6,300,000.00	
			627.93		**9,419,000.00**
From IMU					
10 days accommodation	8 Overseas Students	1,581.10		23,716,500.00	
			1,581.10		**23,716,500.00**

Total Expense			**11,402.65**		**171,039,700.00**
		Saldo	**53.45**		**801,800.00**